Exercise 14

For the following exercises, sketch the parametric equations by eliminating the parameter. Indicate any asymptotes of the graph.

$$x = 4 + 2\cos\theta, \quad y = -1 + \sin\theta$$

Solution

Solve each of the equations for $\cos\theta$ and $\sin\theta$.

$$\frac{x-4}{2} = \cos\theta, \quad y + 1 = \sin\theta$$

Square both sides of each equation and add the respective sides together.

$$\left(\frac{x-4}{2}\right)^2 + (y+1)^2 = \cos^2\theta + \sin^2\theta$$

$$\frac{(x-4)^2}{4} + \frac{(y+1)^2}{1} = 1$$

This is an ellipse centered at $(4, -1)$ with the major axis in the x-direction and the minor axis in the y-direction. Below is a plot of the parametric equations for $0 \le t \le 2\pi$.